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THREE-DIMENSIONAL EDDY CURRENT CALCULATION IN MULTIPLY CONNECTED REGIONS BY USING THE MAGNETIC VECTOR POTENTIAL

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SUMMARY

There are two methods of using the magnetic vector potential for three-dimensional eddy current calculations. The first method uses the continuous magnetic vector potential which accompanies a scalar potential and has the advantage that no cutting is necessary for the multiply connected region problem. The second method uses the discontinuous magnetic vector potential which accompanies no scalar potential and has the disadvantage that cutting is necessary for the multiply connected region problem. In this paper a formulation using the continuous magnetic vector potential and accompanying scalar potential is given, together with computed results for three three-dimensional multiply connected eddy current problems.

1. INTRODUCTION

Consider, for example, the torus problem shown in Figure 1. If the magnetic vector potential is allowed to be discontinuous across the surface of the torus, then the eddy current problem associated with this torus has infinitely many solutions, since the magnetic vector potential $\mathbf{A} = (A_r, A_\theta, A_z) = (0, c/r, 0)$, $r_1 \leqslant r \leqslant r_2$, where c is an arbitrary constant, satisfies curl $\mathbf{A} = 0$, div $\mathbf{A} = 0$ and $\partial \mathbf{A}/\partial t = 0$, $r_1 < r < r_2$, and $\mathbf{n} \cdot \mathbf{A} = 0$ on the surface of the torus, where $\mathbf{n}$ is the unit normal vector to the surface of the torus. Therefore, in order to obtain the unique solution, cutting should be introduced to make the torus simply connected.

In this paper a formulation using the continuous magnetic vector potential and accompanying scalar potential is presented for three-dimensional multiply connected eddy current calculations. Computed results are also presented for two transient FELIX problems[1] and a time-harmonic problem. The calculations were carried out by using the finite difference and boundary integral equation methods.

2. FORMULATION

Assumptions

The space is assumed to be composed of two multiply connected regions as shown in Figure 2. One region is a conducting region which is bounded and the other region is an unbounded free-space region bounding the conducting region. The conductivity and permeability of the conductor are assumed to be constant. (There is no loss of generality in this assumption, since when they

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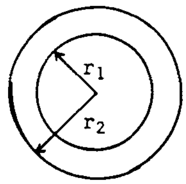


Figure 1. Torus

vary in the conductor, the conductor may be considered to be made up of subregions in which they are constant.) Finally, the displacement current is assumed to be negligible.

Maxwell's equations

$$\text{curl}\,\mathbf{H} = \mathbf{J}, \qquad \text{div}\,\mathbf{B} = 0, \qquad \text{curl}\,\mathbf{E} = -\partial\mathbf{B}/\partial t. \tag{1}$$

Constitutive relations

$$\mathbf{B} = \mu\mathbf{H}, \qquad \mathbf{J} = \sigma\mathbf{E}, \tag{2}$$

where μ and σ are scalar quantities.

Magnetic vector potential and accompanying scalar potential

$$\mathbf{B} = \text{curl}\,\mathbf{A}, \qquad \mathbf{E} = -\partial\mathbf{A}/\partial t - \text{grad}\,\xi, \tag{3}$$

where the first equation follows from the second equation of (1), and the second equation follows from the third equation of (1) and the first equation of (3).

Field equations in terms of **A** and ξ

In the conductor:

$$\text{div}(\text{grad}\,\mathbf{A}_1) - \mu\sigma(\partial\mathbf{A}_1/\partial t + \text{grad}\,\xi) = 0, \tag{4}$$

$$\text{div}(\text{grad}\,\xi) = 0. \tag{5}$$

In free space:

$$\text{div}\,(\text{grad}\,\mathbf{A}_2) + \mu_0\mathbf{J}_0 = 0. \tag{6}$$

At the interface:

$$\mathbf{A}_1 = \mathbf{A}_2, \qquad \mathbf{n}\times\text{curl}\,\mathbf{A}_1/\mu = \mathbf{n}\times\text{curl}\,\mathbf{A}_2/\mu_0,$$

$$\mathbf{n}\cdot\text{grad}\,(\mathbf{n}\cdot\mathbf{A}_1) = \mathbf{n}\cdot\text{grad}\,(\mathbf{n}\cdot\mathbf{A}_2), \tag{8}$$

$$\mathbf{n}\cdot(\partial\mathbf{A}_1/\partial t + \text{grad}\,\xi) = 0 \tag{9}$$

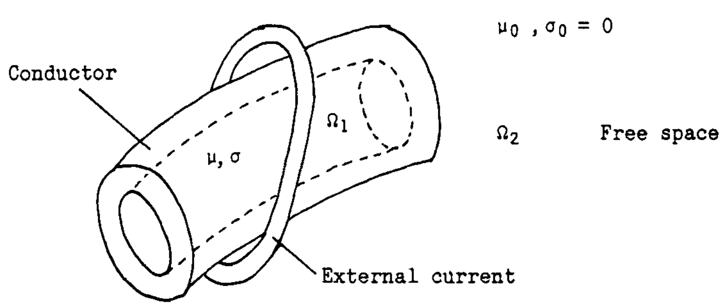


Figure 2. Geometry

Boundary condition

$$\mathbf{A}_2(\mathbf{r}, t) = O(1/|\mathbf{r}|^2) \quad \text{at infinity.}^2 \tag{10}$$

Initial conditions

$$\mathbf{A}_1(\mathbf{r}, 0) = 0, \qquad \mathbf{A}_2(\mathbf{r}, 0) = 0, \qquad \xi(\mathbf{r}, 0) = 0. \tag{11}$$

The system of equations (4)–(11) determines $\mathbf{A}_1(\mathbf{r}, t)$ and $\mathbf{A}_2(\mathbf{r}, t)$ uniquely, and determines $\xi(\mathbf{r}, t)$ to within a constant.[3] (The constant may be set arbitrarily since only grad ξ is needed.)

In order that the solution to the system be the solution to the Maxwell equations (1), $\mathbf{A}_1$ and $\mathbf{A}_2$ should satisfy the Coulomb gauge: div $\mathbf{A} = 0$. This is verified in Reference 3. Using the Coulomb gauge and the identity curl (curl $\mathbf{A}$) = grad (div $\mathbf{A}$) − div (grad $\mathbf{A}$) gives

$$\operatorname{curl}\,[(1/\mu)\operatorname{curl}\mathbf{A}_1] + \sigma(\partial\mathbf{A}_1/\partial t + \operatorname{grad}\xi) = 0 \quad \text{in } \Omega_1,$$

$$\operatorname{curl}\,[(1/\mu_0)\operatorname{curl}\mathbf{A}_2] = \mathbf{J}_0 \quad \text{in } \Omega_2. \tag{12}$$

Equations (12) are the Maxwell equations in terms of the magnetic vector potential and accompanying scalar potential.

Remark. Except for the axisymmetric case, the scalar potential ξ does not vanish since the normal component of the continuous magnetic vector potential is not null at the interface.

3. COMPUTED RESULTS

The formulation using the continuous magnetic vector potential and accompanying scalar potential was first applied to the FELIX short-cylinder problem.[1] This problem consists of a hollow aluminium cylinder placed in a uniform magnetic field. The magnetic field is perpendicular to the axis of the cylinder and decays exponentially with time. The geometry is shown in Figure 3. The resistivity of aluminium is $3{\cdot}94 \times 10^{-8}$ Ω m and the time constant of the exponential

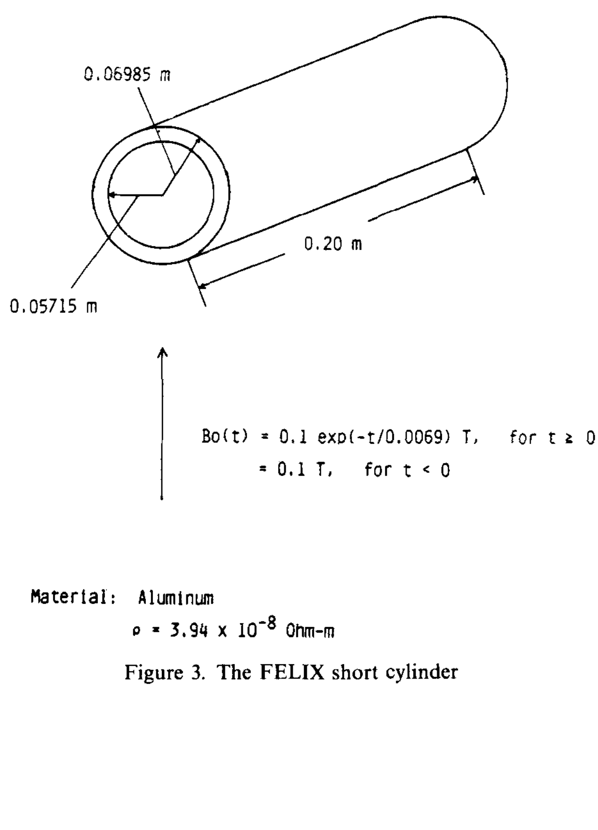


Figure 3. The FELIX short cylinder

decay is 0·0069 s. The applied magnetic field in the y-direction is uniform in space: $B_y(t) = 0{\cdot}1$ T for time $t < 0$ and $B_y(t) = 0{\cdot}1\exp(-t/0{\cdot}0069)$ T for $t \geq 0$.

The finite difference method in cylindrical co-ordinates was applied to the hallow aluminium cylinder and the boundary integral equation method in rectangular co-ordinates to the free space. The boundary integral equation is written as

$$\tfrac{1}{2}\mathbf{A}_2(\mathbf{r},t) = \mathbf{A}_0(\mathbf{r},t) - \int_\Gamma \mathbf{A}_2(\mathbf{r}',t)(\partial/\partial n')(1/4\pi|\mathbf{r}-\mathbf{r}'|)\mathrm{d}s',$$
$$+\int_\Gamma [\partial\mathbf{A}_2(\mathbf{r}',t)/\partial n'](1/4\pi|\mathbf{r}-\mathbf{r}'|)\mathrm{d}s', \quad \mathbf{r}\in\Gamma, \tag{13}$$

where $\mathbf{A}_2 = (A_{2x}, A_{2y}, A_{2z})$, $\mathbf{A}_0 = (A_{0x}, A_{0y}, A_{0z})$, $A_{0x}(\mathbf{r},t) = A_{0y}(\mathbf{r},t) = 0$, $A_{0z}(\mathbf{r},t) = -0{\cdot}1\exp(-t/0{\cdot}0069)\,x$, $t \geq 0$, $\mathbf{r} = (x, y, z)$, and Γ denotes the interface.

The discretization for the boundary integral equation is shown in Figure 4. The computation was carried out for an eighth of the geometry since there is an eightfold symmetry in this problem.

The computed results of the total eddy current and the eddy current distribution are shown in Figures 5 and 6 respectively.

The computed peak value of the total eddy current is attained at $t = 0{\cdot}008$ s (see Figure 5), while the experimental peak value of the induced field B_y on the axis at $z = 0$ is also attained at $t = 0{\cdot}008$ s.[4] Therefore the computation may be considered to be correct.

Computation details: number of finite difference nodes, 90; number of elements for boundary integral equation, 78; number of unknowns, 594; element type, rectangular and zero-order interpolation; time solution method, explicit forward difference; time step, 0·0001 s; number of time steps, 200; solution method for the Laplace equation (div (grad ξ) = 0), Gauss–Seidel method; computer used, IBM 3033; CPU time, 60·0 s.

Secondly, the formulation was applied to the FELIX brick problem.[1] This problem consists of a rectangular aluminium brick with a rectangular hole through it, placed in a uniform field. The magnetic field is perpendicular to the faces with the hole and decays exponentially with time. The

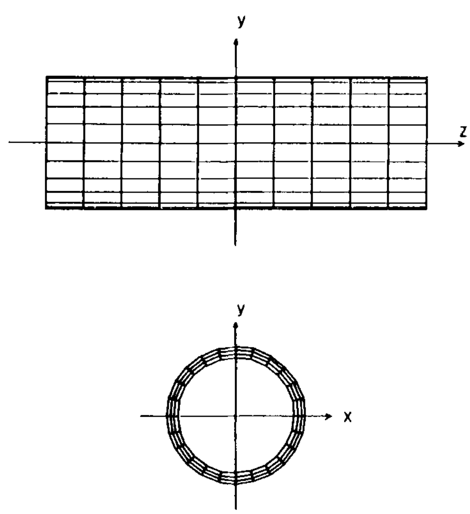


Figure 4. Discretization for the boundary integral equation

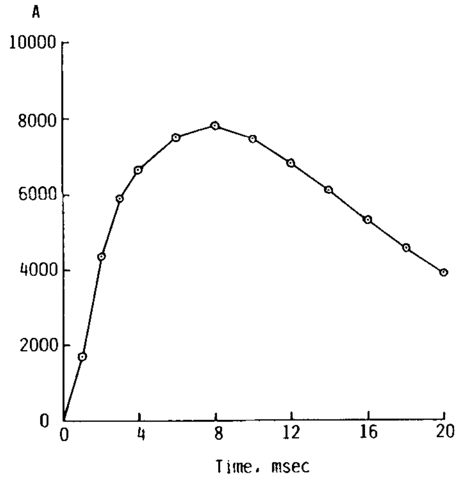


Figure 5. Total circulating current

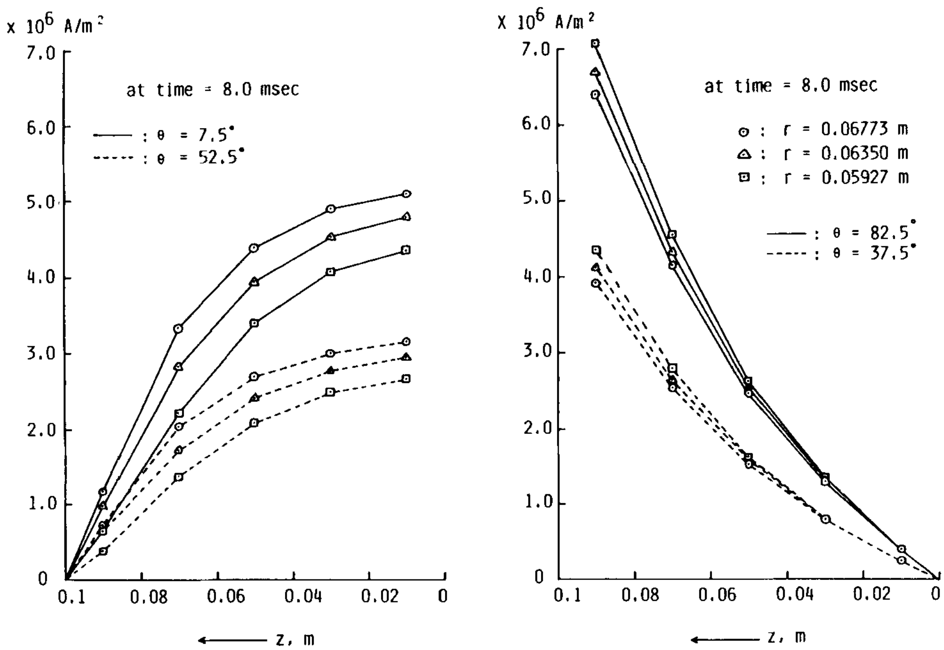


Figure 6. Eddy current distributions

geometry is shown in Figure 7. The resistivity of aluminium is $3{\cdot}94 \times 10^{-8}\,\Omega\,\mathrm{m}$ and the time constant of the exponential decay is 0·0119 s. The applied magnetic field in the y-direction is uniform in space: $B_y(t) = 0{\cdot}1$ Tesla for time $t < 0$ and $B_y(t) = 0{\cdot}1\exp(-t/0{\cdot}0119)$ T for $t \geqslant 0$.

The finite difference and boundary integral equation methods in rectangular co-ordinates were applied to the hollow aluminium brick and the free space respectively.

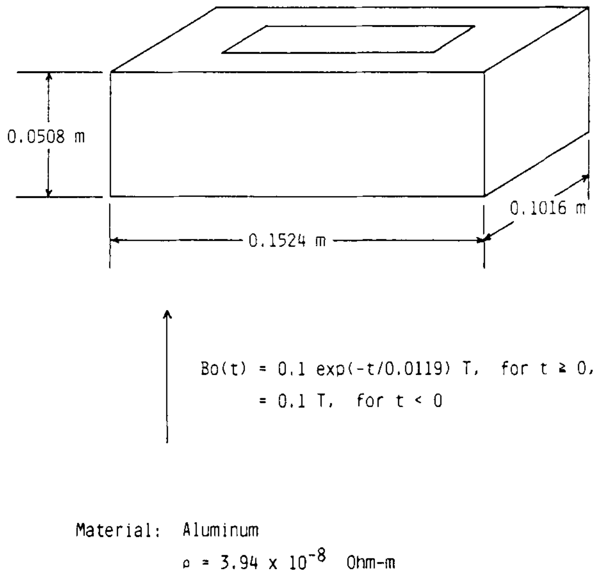


Figure 7. The FELIX brick

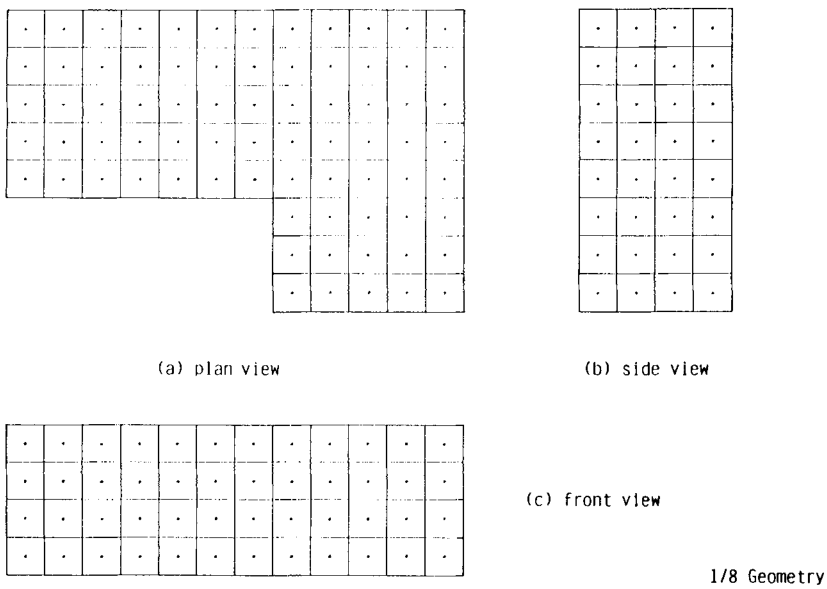


Figure 8. Discretization for the finite difference equation

The discretizations for the finite difference and boundary integral equations are shown in Figures 8 and 9 respectively. The computation was carried out for an eighth of the geometry since there is an eightfold symmetry in this problem.

The computed result of the total circulating current is shown in Figure 10.

This result coincides well with the computed results by other methods.[5]

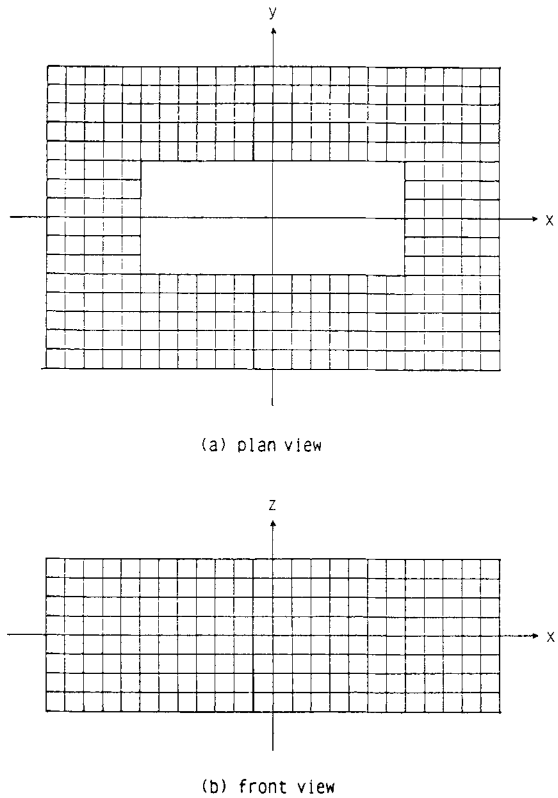


Figure 9. Discretization for the boundary integral equation

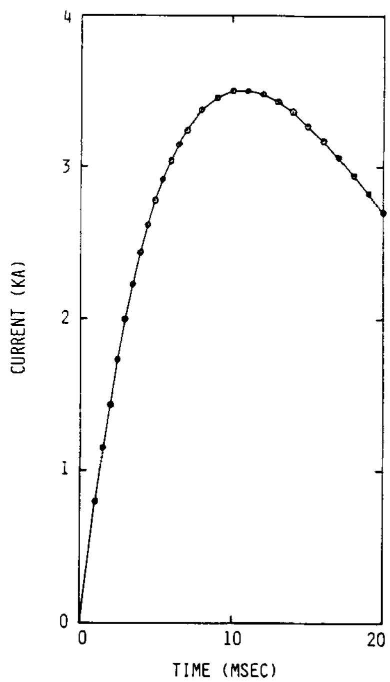
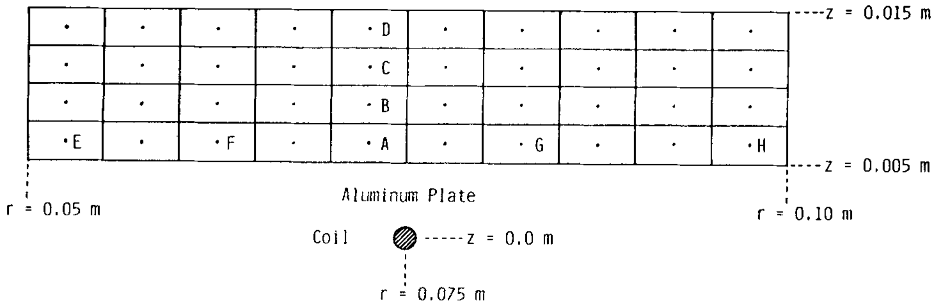


Figure 10. Total circulating current



Physical Properties

Electrical resistivity	:	3.94×10^{-8} Ohm-m
Magnetical permeability	:	$4\pi \times 10^{-7}$ Henry/m
Frequency of Exciting Current	:	50.0 Hz

Figure 11. Nodes for the finite difference equation

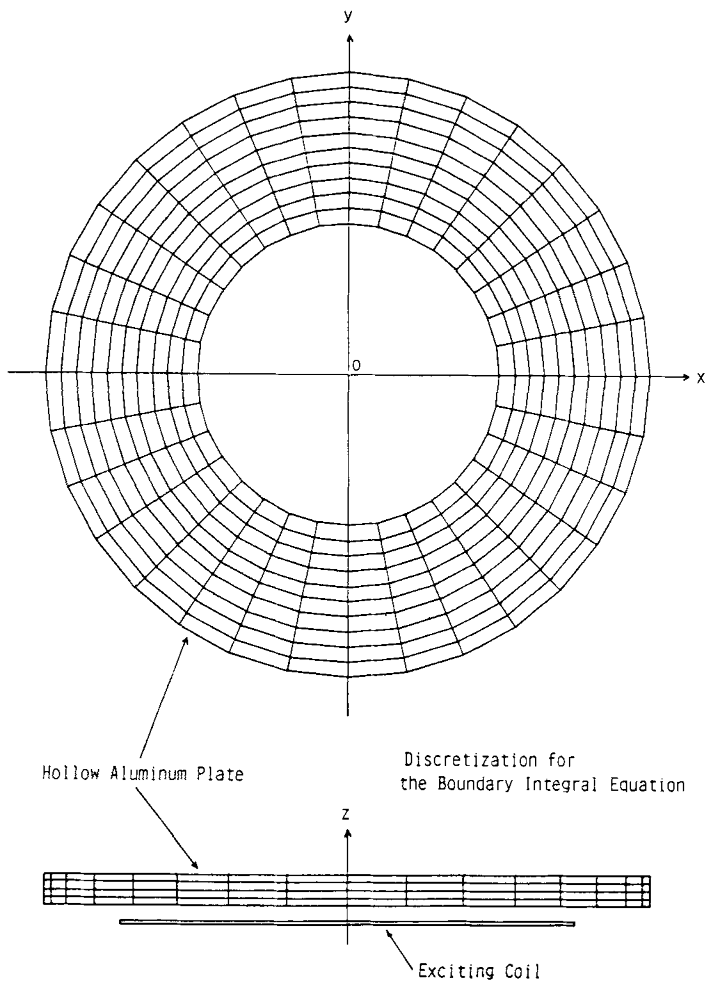


Figure 12. Discretization for the boundary integral equation

Computation details: number of finite difference nodes, 300; number of BIE elements, 195; number of unknowns, 1785; element type, cube for FD and square and zero-order interpolation for BIE; time solution method, explicit forward difference; time step, 0·00005 s for $0 \leqslant t < 0{\cdot}002$ s, and 0·0001 s for $t \geqslant 0{\cdot}002$ s; number of time steps, 220; solution method for the Laplace equation, Gauss–Seidel method; computer used, HP9000-360CH; CPU time, 290 min.

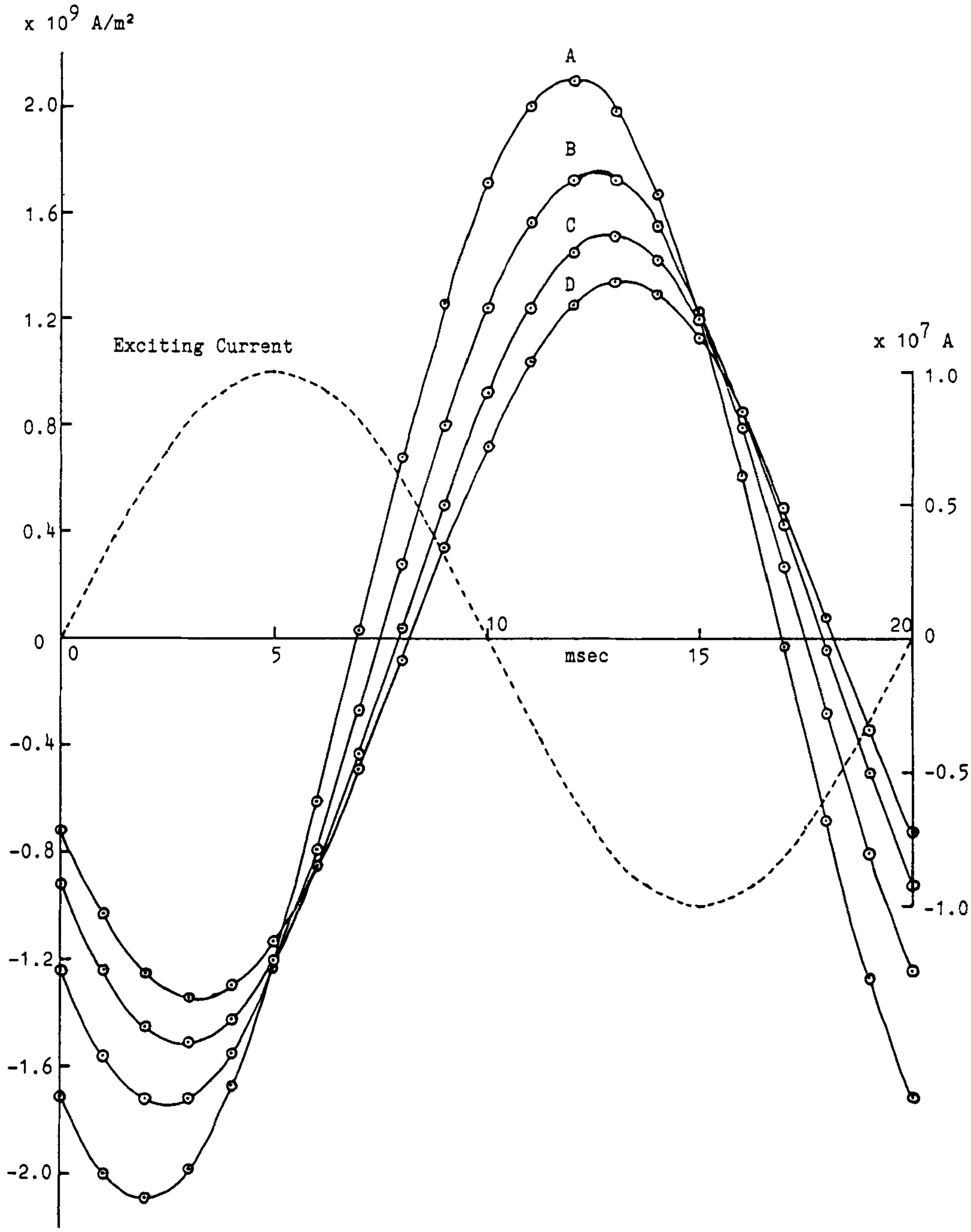


Figure 13. Eddy currents (vertical) in the hollow aluminium plate

Thirdly, the formulation was applied to a time-harmonic problem. This problem consists of a circular aluminium plate with a circular hole through it, placed above a one-turn coil excited time-harmonically. The magnetic field is axisymmetric and varies time-harmonically with a frequency of 50 Hz. The geometry is shown in Figure 11. The resistivity of aluminium is $3{\cdot}94 \times 10^{-8}\ \Omega$ m. The coil current is $10^7 \cos(100\pi t)$ A.

The finite difference equation in cylindrical co-ordinates and the boundary integral equation in rectangular co-ordinates were applied to the hollow aluminium plate and the free space respectively.

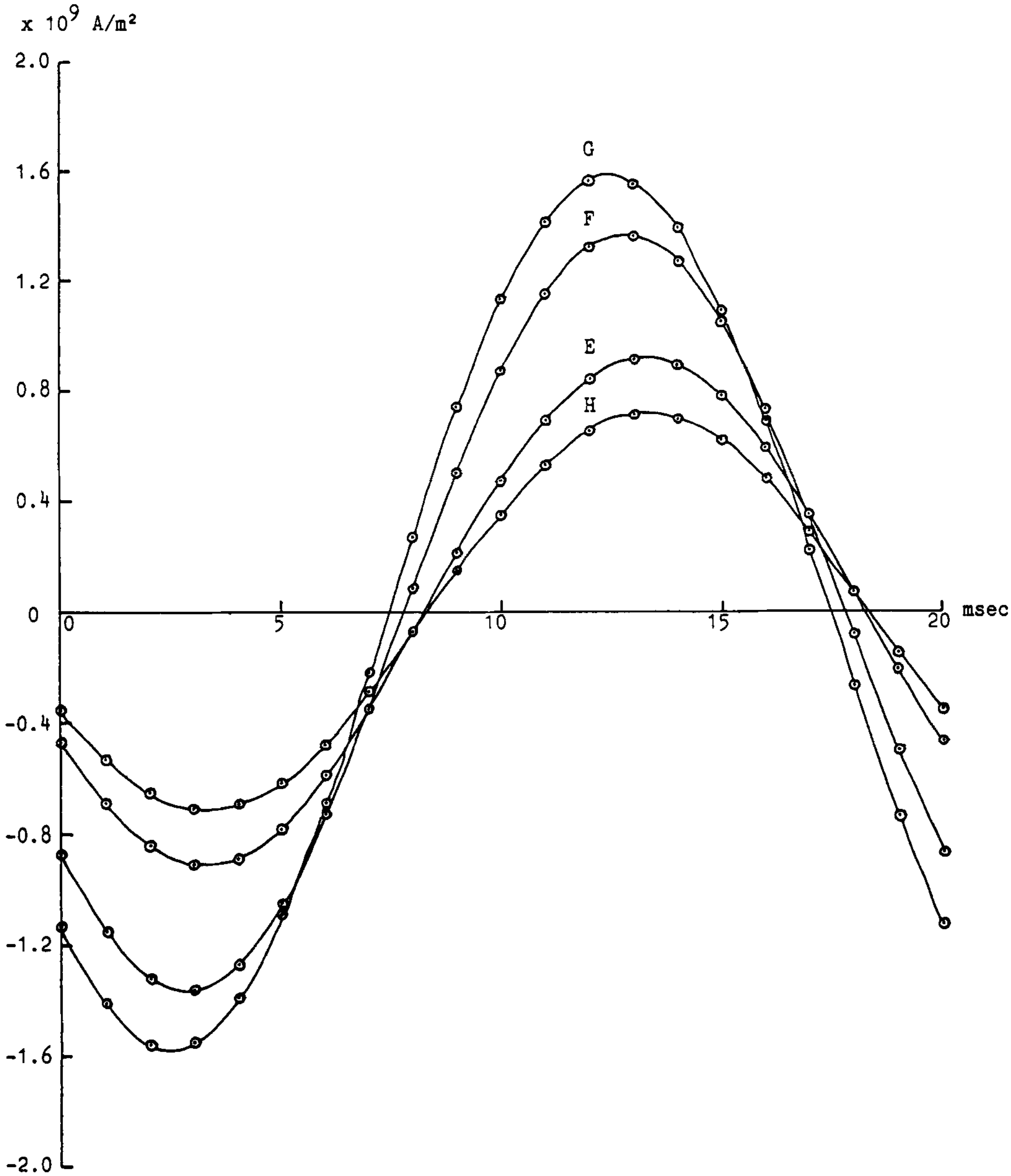


Figure 14. Eddy currents (radial) in the hollow aluminium plate

The discretizations for the finite difference and boundary integral equations are shown in Figures 11 and 12 respectively. The computation was carried out for a cross-section of the geometry since there is an axisymmetry in this problem.

The computed results of the eddy current distributions in the vertical and radial directions are shown in Figures 13 and 14 respectively.

The results coincide well with the computed results by the streamfunction method.[6]

Computation details: number of FD nodes, 40; number of BIE elements, 896; number of unknowns, 68; element type, fan-shaped and zero-order interpolation for BIE; time solution method, explicit forward difference; time step, 0·00002 s; number of time steps, 4000; computer used, HP9000-360CH; CPU time, 13·5 min. (Solving the Laplace equation is not necessary since no accompanying scalar potential exists in this problem.)

4. CONCLUSIONS

In this paper a method using the continuous magnetic vector potential and accompanying scalar potential is presented for three-dimensional multiply connected eddy current calculations. This method has the advantage that it has no topological problem, while the method using the discontinuous magnetic vector potential accompanying no scalar potential is forced to introduce cutting to the multiply connected problem. From the computed results, the formulation given in this paper may be considered to be correct.

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